

Physics 226: Problem Set #2
Due in Class on Thurs Sept 15, 2016

1. We saw in class that the *Central Limit Theorem* tells us that the distribution of the sum (or average) of a large number of independent, identically distributed measurements will be approximately normal, regardless of the underlying distribution (subject to the condition that mean and variance of the underlying distribution are not infinite). We'll see how this works for the simplest pdf, a random variable x uniformly distributed:

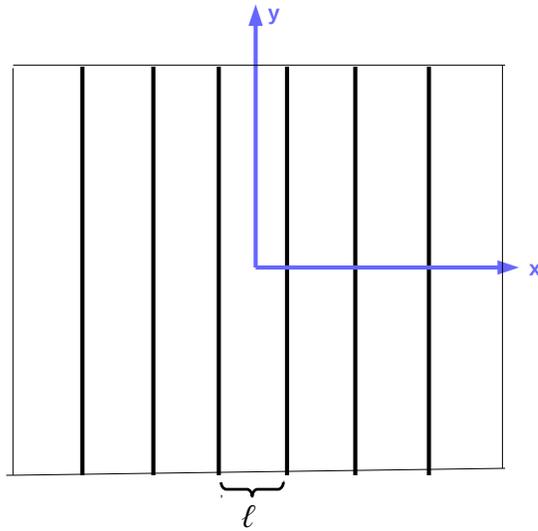
$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

- (a) Use the definitions of mean and sigma (given on page 6 of the slides for lecture 4) to calculate the mean μ and the variance σ of the distribution
- (b) Let $a = 0$ and $b = 1$. Using your favorite random number generator, generate 1000 random numbers. Calculate the mean and sigma of these numbers and verify that the results are consistent with your result from part (a).
- (c) Make a histogram with 100 bins where the lower edge of the first bin is at $x = 0$ and the upper edge of the last is at $x = 1$. Fill your histogram with the random numbers you generated in part (b).
- (d) Now suppose you make an ensemble of 1000 pseudoexperiment where each pseudoexperiment consists of N uniformly distributed random numbers. For each pseudoexperiment, define the measurement S to be

$$S \equiv \frac{1}{N} \sum_1^N x_i$$

Make histograms of S with the same x -axis as in part (c) for the cases $N = 2$, $N = 5$ and $N = 10$. Determine the mean and the σ of the distributions displayed in these histograms. In each case, compare the σ you obtain to what you would predict if you assumed the experiments followed a normal distribution.

2. In this problem and the next we will study how the position resolution of a detector depends upon the properties of that detector. For our example, we will consider a silicon strip detector. We will describe our detector as a plane segmented into strips, each of width ℓ . When a track passes through the plane, it deposits energy in the detector and that energy is collected using charge sensitive amplifiers (one per strip). You may assume that the incident track is normal to the silicon plane. Looking down on the strip detector (so that the incident tracks are traveling into the page), the detector looks like this:



The position $x = 0, y = 0$ is taken to be the center of the middle strip.

- (a) Suppose all the energy is deposited in a single strip (the strip the track passes through). Find an expression for the position resolution of the detector as a function of ℓ . The position resolution is defined to be $\sigma_x = \sqrt{\text{var}[(x_{meas} - x_{true})]}$ where x_{true} is the position where the track actually hit the detector. Because we only know which strip is hit, in this example x_{meas} is the center of the strip that is hit.
- (b) Suppose that the charge deposited in our detector spreads out due to physical effects such as diffusion. It is possible for more than one strip to register a signal. Assume in this part that our electronics is

binary (ie registers a 1 if the deposited energy on the strip is above a specified threshold and 0 otherwise) Assume the threshold on the electronics is such that particles hitting within a distance of $\pm\ell/3$ of the center of the strip only register on a single strip while all particles hitting further from the strip center register on two strips. What is the position resolution now? (Here, if only one strip is hit, x_{meas} is the center of the strip. If two strips are hit, then x_{meas} is the common edge of the two hit strips).

3. In problem 2, it was possible to calculate the position resolution analytically. In cases where the detector response is more complicated, this may not be the case. Typically, particle physicists model detector performance using Monte Carlo simulations. In problem set # 1 you used a simulation to describe the response of a calorimeter to an electromagnetic shower. In this problem, you will write a simple simulation to determine the position resolution of a silicon detector.

(a) Let's begin by reproducing the analytic results obtained in problem 2. Consider a silicon strip detector that consists of 7 strips of width ℓ . Assume that the incident particles have a uniform distribution in x with $-\ell/2 < x < \ell/2$ and all have $y = 0$. Generate 10,000 such particles for the case described in part 2(a) and for the case described in part 2(b). For each case, make a histogram of $(x_{meas} - x_{true})$ and verify that the resolution is consistent with that obtained in problem 2.

(b) Now, let us replace our binary electronics from part 2(b) with analog electronics (so that the magnitude of the charge deposited on the strip is recorded). We will model the transverse spreading of the charge from our incident track using a Gaussian distribution with width σ_M :

$$f(x)dx = \frac{1}{\sigma_M\sqrt{2\pi}} \exp(-(x - x_0)^2/2\sigma_M^2)$$

where $f(x)$ is the charge deposited between position x and $x + dx$ and x_0 is the point where the track hits the detector. Assume that the total energy deposited by each track is 1 MIP (a MIP is the energy

deposited by a single minimum ionizing particle), that our analog electronics has a threshold of 0.2 MIP and that $\sigma_M = \ell$. Also assume that the electronics has an intrinsic noise contribution $\sigma_N = 0.05$ MIP. (This means that the measurement of the charge on each strip is modified by adding a noise contribution that is distributed according to a Gaussian with mean 0 and variance σ_N^2 .) Generate 10,000 particles and simulate the response of this silicon strip detector to these particles. From this simulation determine the position resolution of the silicon detector. Assume that in the analysis of these data the measured position of the particle is:

$$x_{meas} = \sum_{i=strips} q_i x_i$$

where the index i is the strip number, q_i is the measured charge on the strip (set to zero for strips with charge below threshold) and x_i is the position of the center of strip i .

Note: To do this problem, you will need to use a mathematical package that allows for the evaluation of error functions.