Physics 226: Problem Set #1
Due Thurs Sept 6, 2018

Some of the problems below require numerical calculations (integrals, random number generation, etc). You are free to use your favorite mathematical software package and your favorite computer language to do the problems. My solutions will use Root and C++. I've posted on our web page information about Root, including an example of how to write a simple Monte Carlo program to generate particle decays. You might find this example helpful in doing Problem 3 below.

In doing these problems, you will need to know certain physical constants (particle lifetimes, radiation lengths of specific materials, etc). You can find everything you need on the Particle Data Group (PDG) web site:

http://www-pdg.lbl.gov/

Take some time to learn to navigate this site. We'll be using it extensively.

1. *Relativistic expressions relevant for accelerator performance* (30 points)

   (a) Explicitly derive the expressions for the center-of-mass energy for the collider and fixed target caes *not* assuming that the masses of the colliding particles are small.

   (b) What approximations do you have to make to derive the simpler formulae usually quoted:

   $$E_{cm}^{collider} = \sqrt{4E_1E_2} \text{ and } E_{cm}^{fixed\ target} = \sqrt{2m_{target}E_{beam}}$$

   (c) The Babar experiment (which ran at SLAC until Spring 2008) and the recently upgraded Belle experiment at KEK in Japan study B mesons produced in $\Upsilon(4s)$ decays through the process $e^+e^- \to \Upsilon(4s) \to B\overline{B}$. Both accelerators were designed to operate at a center-of-mass energy of 10.56 GeV (corresponding to the $\Upsilon(4s)$ mass). Both accelerators are "asymmetric colliders," where the energies of the e^+ and e^- beams are not the same. This asymmetry means that the center-of-mass is boosted along the beam direction and therefore the produced B mesons are moving, making it possible to measure their lifetime by looking at the distance they travel before decaying.

i. For a boost of $\beta\gamma = 0.56$ along the e^+ direction of motion, determine the required energies of the e^+ and e^-.

ii. Determine the average distance between the B production point and its decay point in the LAB. (Note: You can find the value of the B meson lifetime on the PDG website. The charged and neutral B mesons have almost the same lifetime. Use the B^0 lifetime for this problem. Also, note that in the center-of-mass energy of the B-factories is very close to twice the B mass. You may therefore make the approximation that the B mesons are produced at rest in the center-of-mass.)

iii. For the two body decay $B^0 \to \pi^+\pi^-$, determine the range of the possible LAB momenta of the two pions. Note: For this calculation, you may make the approximation that the pion is massless.

2. *Particle ID using a time-of-flight detector* (30 points)
The CDF experiment had a time-of-flight (TOF) system. The purpose of the TOF was to distinguish $\pi^\pm$ from $K^\pm$. The system had a time resolution $\sigma_t = 100$ ps and was located 1.4 m from the point where the π and K were created.

(a) Let's begin by deriving an expression for the difference in flight time for two relativistic particles of masses m_1 and m_2 with the same momentum p that travel a distance ℓ. (Hint: start with the relativistic expression $\beta = p/E$ and Taylor expand the energy E for the case where $m \ll p$).

(b) Assume that you have equal populations of π and K. Using the expression you have derived, find the maximum momentum for which you can identify a K at better than the 90% confidence level. (If you are not familiar with the concept of a "confidence level" see the review of statistics in the PDG. Warning: this is a single-sided confidence level.) You may assume all errors are Gaussianly distributed.

(c) Suppose a factor of 3 more π than K are produced at a momentum of 1.5 GeV. What cut on time-of-flight must be applied in order to

produce a sample of candidate kaons with a purity P of 80% ($P \equiv n_K/(n_K + n_\pi)$)? What is the efficiency ($\epsilon = n_K^{passing}/n_K^{total}$) of this cut for K at this momentum? Note, you will have to solve this last part numerically. Feel free to use Mathematica or your favorite mathematics package to do this.

3. *A Monte Carlo Model of Electromagnetic Showers* (40 points)
In order to understand how their detectors respond, particle physicists use complex simulations. These simulations take as their input Monte Carlo generated events, propagate the particles in these events through a model of the detector and simulate the response. The output of the simulation is written in the same format as the real data and is used to understand the experiment's "acceptance" and resolution. Proper modeling of the detector response requires detailed understanding of the physics of particle interactions with matter. Today, most experiments use a simulation toolkit, called Geant4, to model this physics. In this problem, you will write your own Monte Carlo simulation of electromagnetic showers and use it to describe shower development in the CMS electromagnetic calorimeter (ECAL). A description of the CMS ECAL can be found at:

http://cms.web.cern.ch/news/electromagnetic-calorimeter

Write a Monte Carlo simulation that predicts the longitudinal development of an electromagnetic shower in the CMS ECAL. The final "answer" should be a plot that looks roughly like the black circles in Figure 33.20 of the PDG Review of the *Passage of Particles Through Matter*, but where the horizontal axis is the distance in cm from the front face of the calorimeter and the vertical axis is the average number of *charged* particles crossing a plane at that distance. To understand how the shower development depends on energy, make this plot for $E = 1$ GeV and $E = 10$ GeV electrons and compare the distance at which the maximum occurs. In order to keep the statistical uncertainties small, generate 1000 events at each energy.

You will need to make a number of simplifying assumptions in your model:

- Describe the calorimeter as a uniform crystal of lead tungstate, 23 cm deep. Assume electrons hit the front face of the crystal with fixed energy E and normal to the surface.
- Real EM showers develop in 3-dimensions, for this problem use a 1-dimensional model and ignore the transverse spreading of the shower.
- Electrons lose energy by bremsstrahlung. The mean distance over which a high energy electron loses all but $1/e$ of its energy is called the radiation length X_0. The bremsstrahlung spectrum of the emitted photons is peaked at low photon energy. While the true bremsstrahlung process is continuous, for this problem make the *unrealistic* approximation that the energy loss is a discrete process that occurs at random positions x along the electron trajectory. In other words, the probability of a discrete bremmstahlung occuring in distance dx is assumed to be
$$dP \equiv \frac{dN}{N} = -\frac{dx}{X_0}$$
To simplify the calculation, also make the *unrealistic* assumption that whenever the bremsstahlung occurs the energy is divided equally between the the outgoing electron and photon.
- When charged particles travel through matter, they lose energy via ionization. The distribution of energy loss per unit distance is a Landau distribution, with a mean that depends on the particle's velocity. Assume for this problem that the ionization energy loss per cm is constant, with the value for lead tungstate taken from the PDG *Atomic and nuclear properties of materials*. If a charged particle (e^+ or e^-) loses enough energy via ionization to stop in the crystal, then it will just be absorbed in the material. (In the real world, charged particles have a Bragg peak in their energy loss as they stop. We will ignore that effect here).
- Photons lose energy via Compton scattering, photo-nuclear interactions and pair production. Assume for this problem that pair production is the only process that matters and that the probability of

pair production occuring distance between x and $x + dx$ is

$$dP = \frac{dx}{9/7X_0}$$

Also *unrealistically* assume that the e^+ and e^- produced always share the energy of the photon equally. Make the approximation that the electron is massless.

Note: if you are not familar with Monte Carlo methods, this problem will be difficult. Please come to section and ask questions.