Lecture 6: Introduction to Symmetries and Conservation Laws

September 11, 2018

Overview

- SM: elegant theoretical construct based on principles of gauge invariance and renormalizability
  - ▸ Principles essential for constructing consistent theory
  - ▸ Don't tell us what the spectrum of particles or the interactions are.
  - ▸ Experimental input required
- This week: develop structure of the matter fields of SM and the nature of interactions based on experimental observations
- Rely heavily on concepts of symmetry and invariance:
  - ▸ Strong and Weak interactions don't have classical analogs
  - ▸ Unlike EM, don't have a classical Hamiltonian or Lagrangian as a starting point for defining the theories
  - ▸ Use observed symmetry properties to help deduce the the form of the Lagrangian
  - ▸ This, together with local gauge invariance, defines the theory

Symmetries and Conservation Laws

- Symmetry of H: Operator R leaves H unchanged

$$R^{-1}H(t)R = H(t)$$

- Relationship between symmetries and conservation laws:

$$i\frac{dQ}{dt} = i\frac{\partial Q}{\partial t} + [Q, H]$$

If operator has no explicit time dependence

$$[Q, H] = 0 \implies <Q> \text{ is conserved}$$

- Conserved quantum #'s are associated with operators that commute with H (Noether's Theorm)
- Most common examples:
  - ▸ space-time invariance (translations) $\iff$ energy-momentum conservation
  - ▸ space-time invariance (rotations) $\iff$ angular momentum conservation

Nature of H_{int} Determines the Symmetry Properties

- While space-time invariance is a symmetry of all known interactions, many other particle physics symmetries hold only for certain H_{int}
  - ▶ Some conservation laws apply only for certain interactions
  - ▶ Strong interactions exhibit the largest amount of symmetry
  - ▶ EM comes next
  - ▶ Weak even less
- At low energy (where most particle decays happen)
  <center>Strong coupling $>>$ EM $>>$ Weak</center>
- This determines decay hierarchy:
  - ▶ If a strong decay is possible, it will happen first
  - ▶ If symmetry prevents strong decay, EM comes next
  - ▶ Weak comes last

Transformations of Interest in Particle Physics

- Continuous Space-Time Transformations
  - ▸ Translations
  - ▸ Rotations
  - ▸ Extension of Poincare group to include fermionic anticommuting spinors (SUSY)
- Discrete Transformations
  - ▸ Space Time Inversion (Parity=P)
  - ▸ Particle-Antiparticle Interchange (Charge Conjugation=C)
  - ▸ Time Reversal (T)
  - ▸ Combinations of these: CP, CPT
- Continuous Transformations of Internal Symmetries
  - ▸ Isospin
  - ▸ $SU(3)_{\text{flavor}}$
  - ▸ $SU(3)_{\text{color}}$
  - ▸ Weak Isospin

Continuous Space Time Transformations

- Translations
  - ▸ Infinitesmal: $\mathcal{D} = 1 + \delta r \frac{\partial}{\partial r}$
  - ▸ Finite: $\mathcal{D} = e^{ip\Delta r}$
- Rotations
  - ▸ Infinitesmal: $\mathcal{R} = 1 + \delta\phi \frac{\partial}{\partial\phi}$
  - ▸ Finite: $\mathcal{R} = e^{iJ_z\Delta\phi}$

- Symmetries under continuous transformations lead to additive conservation laws!

All interactions are invariant under these global space-time transformations

Intrinsic Spin

- From QM, know particles have intrinsic spin
  - Spin $\frac{1}{2}$: electrons, protons, neutrons
  - Spin 1: photon

- Simple extension of the algebra used for orbital angular momentum
  - For spin-$\frac{1}{2}$ particles

  $$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

  - Transformation: $\psi \to e^{i\vec{\sigma}\cdot\hat{n}\theta/2}\psi$

  $$\begin{aligned} \psi' &\to \psi + \delta\psi \\ \delta\psi &= i\theta\hat{n}\cdot\left(\frac{\vec{\sigma}}{2}\psi\right) \end{aligned}$$

  - θ defines the magnitude of the rotation angle in spin space
  - $\hat{n}$ is the axis of rotation
  - The Pauli matrices $\vec{\sigma}$ are a representation of SU(2)
    - 2×2:' "fundamental representation" of SU(2)

Determining spin of other particles

- Experimentally determined from particle's decays and interactions
- Can tell bosons from fermions by whether than can be singly produced
  - Eg ν is fermion since β-decay $n \to pe^-\overline{\nu}$
- Measure rates or angular distributions to further determine value of spin
  - Eg Spin of the π^+ is 0:

$$pp \to \pi^+ d$$

Principle of Detailed Balance: $|\mathcal{M}_{if}|^2 = |\mathcal{M}_{fi}|^2$

$$\sigma(pp \to \pi^+ d) = (2s_\pi + 1)(2s_d + 1){p_\pi}^2$$
$$\sigma(\pi^+ d \to pp) = \frac{1}{2}(2s_p + 1)^2 {p_\pi}^2$$

($\frac{1}{2}$ due to identical particles in final state)

Discrete Transformations: P, C, T

- These symmetries depend on characteristics of the Lagrangian
- Because EM interaction symmetric under all 3 discrete transformations, we are familiar with them from Quantum Mechanics
- Strong interaction also symmetric under C, P and T separately
- Weak interaction is not:
  - ▸ Maximum violation of P (totally left handed charged current interaction)
  - ▸ Small violation of simulataneous application of C and P ($\approx 10^{-3}$ effect)
  - ▸ All field theories invariant under simultaneous application of C, P and T (CPT theorem)
- Symmetries under discrete transformations lead to multiplicative conservation laws!

$\Rightarrow$ Must test whether each symmetry is respected by each interaction

Parity

- Parity operator defined as spatial inversion

$$
\begin{aligned}
(x, y, z) &\longrightarrow (-x, -y, -z) \\
P(\psi(\vec{r})) &= \psi(-\vec{r})
\end{aligned}
$$

- Repetition of the operations gives $P^2 = 1$
  - P is a unitary operator with eigenvalues ± 1

- If system is an eigenstate of P, its eigenvalue is called the parity of the system

Reminder: Parity

- Something familiar from atomic physics and quantum mechanics:

$$
\begin{aligned}
\psi(r,\theta,\phi) &= \chi(r)Y_{\ell m}(\theta,\phi) \\
&= \chi(r)\sqrt{\frac{2\ell+1)(\ell-m)}{4\pi(\ell+m)!}}P_{\ell m}(\cos\theta)e^{\imath m\phi}
\end{aligned}
$$

- Spatial inversion:

$$
\vec{r} \to -\vec{r} \text{ is equiv to } \theta \to \pi-\theta, \ \phi \to \phi+\pi.
$$

Thus:

$$
\begin{aligned}
e^{\imath m\phi} &\to e^{\imath m(\phi+\pi)} \to (-1)^m \\
P_{\ell m}(\cos\theta) &\to (-1)^{\ell+m}P_{\ell m}(\cos\theta) \\
Y_{\ell m}(\theta,\phi) &\to (-1)^\ell Y_{\ell m}(\theta,\phi)
\end{aligned}
$$

- Spherical harmonics have parity $(-1)^\ell$

More on the Parity Operator

- Define $U_P \equiv P$ such that $U_P \psi(\vec{r}) = \psi(-\vec{r})$
- $U_P^\dagger = U_P = U_P^{-1}$
- How do various operators transform under P?

$$U_P \, \vec{r} \, U_P^{-1} = -\vec{r}$$
$$U_P \, \vec{p} \, U_P^{-1} = -\vec{p}$$
$$U_P \, \vec{L} \, U_P^{-1} = +\vec{L}$$
$$U_P \, \vec{S} \, U_P^{-1} = +\vec{S}$$

Note:

1. Parity is a multiplicative quantum number

$$P(\psi = \phi_a \phi_b) = P(\phi_a) P(\phi_b)$$

2. Spin must be an axial vector since L is an axial vector

Parity and Elementary Particles

- *If* parity is a good symmetry of H_{int}, all elementary particles must be eigenstates of P with eigenvalues ± 1.

- To determine if parity is a good symmetry, see if it's possible to uniquely define eigenstates for each elementary particle (independent of reaction)
  Note: It is not necessarily true that definition be *unique* as long as there is a consistent one

- Experimental Facts:
  - ▸ Both Strong and EM interactions conserve parity
  - ▸ Weak interactions do not

  We'll talk more about this in a few weeks, but notice:
  - ▸ That weak interactions don't conserve P is clear from fact that ν are always left-handed and $\bar{\nu}$ are always right handed.

Elementary Particles Have Intrinsic Parity

- The Photon
  - ▸ Electric current is a vector not an axial vector so $P(\gamma) = -1$
- Dirac Particles
  - ▸ Dirac Eq and definition of vector current require particle and anti-particle to have opposite parity
  - ▸ Since they are always pair produced, it is a matter of convention as to which is $+$ and which is $-$
- Pions
  - ▸ Pions are bosons with spin 0 and three charge states
    $$\pi^+, \pi^0, \pi^-$$
  - ▸ Since bosons, they can be produced singly:
    P can be measured by studying reactions
  - ▸ See next two pages for details

Parity of the Charged Pion

- Study $\pi^- d \to nn$
    - π capture from s-wave (mesonic x-ray spectrum and rate)
    - Spin(d)=1 and Spin(π)=0 and $L = 0$ so $J = 1$ for initial state
    - What are the possibilities for the nn state?

$$L = 0 \qquad S = 1$$
$$L = 1 \quad S = 0, 1, 2$$
$$L = 2 \qquad S = 1$$

    - Fermi statistics: nn w.f. must be anti-symmetric
        Symmetry of wf: $(-1)^\ell (-1)^{s+1}$
    - Only $L = 1, S = 1$ state is possible
    - Thus nn are in a $^3P_1$ state with parity $(-1)^\ell = -1$
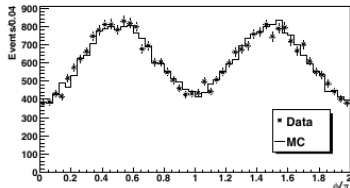   - To determine P of deuteron: p and n have $P = 1$. Also, we know $L = 0$ so deuteron has $P = 1$

$$\Rightarrow \pi^- \text{ has } P = -1 \text{ (pseudoscalar)}$$

Parity of the Neutral Pion

- Main decay mode $\pi^0 \rightarrow \gamma\gamma$
  - But to measure P in this mode, must measure γ polarization
- Instead use $\pi^0 \rightarrow (e^+e^-)(e^+e^-)$ (BR$\sim 10^{-4}$)
  - Look at polarization planes of e^+e^- pairs: Two possible forms

$$\psi \propto (\epsilon_1 \cdot \epsilon_2) \quad = \quad \cos\phi \;\; \text{scalar}$$
$$\psi \propto (\epsilon_1 \times \epsilon_2) \cdot \vec{k} \quad = \quad \sin\phi \;\; \text{pseudoscalar}$$

PRL 100, 182001 (2008)

FIG. 4: Distribution of the angle ϕ, in units of π, between the planes of the two e^+e^- pairs. The solid histogram shows the Monte Carlo expectation for negative parity.

$$\Rightarrow \pi^0 \text{ has } P = -1 \text{ (pseudoscalar)}$$

Charge Conjugation (C)

- C reverses the sign of the charge and magnetic moment and leaves spatial coordinates unchanged
- Maxwell's eq are intrinsically invariant under C
- Strong interactions also conserve C, but weak interactions don't
- C really changes particle $\rightarrow$ antiparticle (thus lepton and baryon number change sign under C)
- C is more difficult to study than P because elementary particles aren't in general eigenstates of C

$$C \left| \pi^+ \right\rangle = \eta \left| \pi^- \right\rangle \neq \pm \left| \pi^+ \right\rangle$$

$$C \left| \pi^0 \right\rangle = \eta \left| \pi^0 \right\rangle$$

with $|\eta|^2 = 1$ so $\eta = \pm 1$

- To find the sign, note that EM fields are produced from charges
- Changing sign of charge changes direction of $\vec{E}$
- Photon has $C = -1$
- Since $\pi^0 \to \gamma\gamma$, π^0 has $C = 1$
- Consequence: $\pi^0 \to 3\gamma$ is forbidden

$$\frac{\pi^0 \to 3\gamma}{\pi^0 \to 2\gamma} < 3.1 \times 10^{-8} \quad 90\% \text{ cl}$$

Although charged particles aren't eigenstates of C, C invariance is still useful for relating reaction rates

$$\mathcal{M}(ab \to cd) = \mathcal{M}(\bar{a}\bar{b} \to \bar{c}\bar{d})$$

Time Reversal (T)

- Operator T turns $t \to -t$
- Tested experimentally for strong interactions by applying principle of detailed balance
- T is also a good symmetry of EM. Small T violations in Weak interaction (more in a few weeks)
- A good test of Time reversal symmetry: the neutron EDM

The CPT Theorm

- All interactions that can be described by quantum field theory are invariant under the combined operations of C, P and T: CPT
- We will see in a few weeks that weak interactions have small CP non-invariance (as well as T invariance)

The Hadronic Spectrum

- 3 generations of leptons (e, μ, τ and respective ν)
- Hundreds of hadrons:
  - ▶ Are they fundamental?
  - ▶ Look for patterns in mass, spin, charge
  - ▶ Rules to relate interaction and decay rates
- Hadrons are composite particles made of quarks
  - ▶ Spectrum of particles analog of period table of elements
  - ▶ α_s large at low q^2 → theory not perturbative
  - ▶ Can't calculate wave functions of the quark bound states
  - ▶ Heavy quark bound states probe QCD potential (week 8)
- In 1960's no one knew whether quarks were real or just mathematical constructs
  - ▶ We'll use modern knowledge to inform our discussion and terminology

Classification of Hadrons (I)

- Mesons (integer spin) vs Baryons (half integer spin)
  - ▶ Baryons must be pair produced, mesons can be produced singly
- Baryons
  - ▶ Earliest examples: p and n
  - ▶ Fact that both appear to see same nuclear force and that the masses are so close together ($m_p = 938.20$ MeV, $m_n = 939.57$ MeV) make it natural to think of them as 2 states of same particle: the nucleon N
  - ▶ Define isospin (with same algebra as spin: SU(2)). Then N has $I = \frac{1}{2}$:

$$
\begin{aligned}
N &\equiv \begin{pmatrix} u \\ d \end{pmatrix} \\
p &= \left| \frac{1}{2} \frac{1}{2} \right\rangle \\
n &= \left| \frac{1}{2} - \frac{1}{2} \right\rangle
\end{aligned}
$$

- Mesons
  - Earliest example: The pions
  - Three charges π^+, π^0, π^- so $I = 1$:

$$\Pi \equiv \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

$$\pi^+ = |11\rangle$$
$$\pi^0 = |10\rangle$$
$$\pi^- = |1-1\rangle$$

- Note: For nucleon $Q = I_z + \frac{1}{2}$ while for pion $Q = I_z$
  These are special cases of a more general rule we'll get to soon

- Use isospin to relate reaction rates
- One matrix element per value of I_{Tot}
- $I = 1 \otimes \frac{1}{2} \Rightarrow I_{Tot} = \frac{3}{2}$ or $\frac{1}{2}$ so two indep matrix elements:

$$\mathcal{M}_{\frac{1}{2}} \equiv \left\langle \frac{1}{2} \middle| H \middle| \frac{1}{2} \right\rangle \quad \mathcal{M}_{\frac{3}{2}} \equiv \left\langle \frac{3}{2} \middle| H \middle| \frac{3}{2} \right\rangle$$
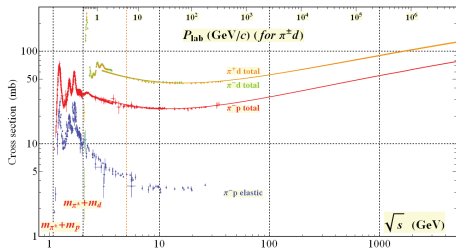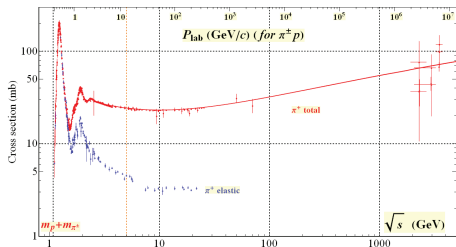
- Examples of decomposition

$$p\pi^+ = \left| \frac{3}{2} \frac{3}{2} \right\rangle$$

$$p\pi^0 = \sqrt{\frac{2}{3}} \left| \frac{3}{2} \frac{1}{2} \right\rangle - \sqrt{\frac{1}{3}} \left| \frac{1}{2} \frac{1}{2} \right\rangle$$

- Thus

$$\sigma(\pi^+ p \to \pi^+ p) \sim \left| \mathcal{M}_{\frac{3}{2}} \right|^2$$

$$\sigma(\pi^+ n \to \pi^+ n) \sim \left| \frac{1}{3} \mathcal{M}_{\frac{3}{2}} + \frac{2}{3} \mathcal{M}_{\frac{1}{2}} \right|^2$$

$$\sigma(\pi^- p \to \pi^0 n) \sim \left| \frac{\sqrt{2}}{3} \mathcal{M}_{\frac{3}{2}} - \frac{\sqrt{2}}{3} \mathcal{M}_{\frac{1}{2}} \right|^2$$

- Large bumps: "resonances"
- Eg: near 1236 MeV
  - Width ~ 120 MeV $\Rightarrow$ short lifetime: $\Delta E \Delta t \sim \hbar$:

  $$\Delta t \quad \sim \quad \frac{\hbar}{\Delta E}$$
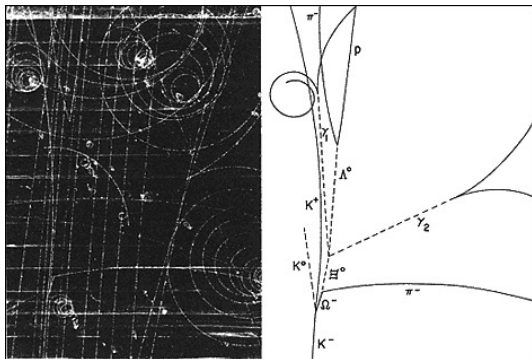 $$\sim \quad \frac{6.58 \times 10^{-22} \text{ MeV s}}{120 \text{ MeV}}$$
  $$\sim \quad 5 \times 10^{-24} \text{ s}$$

- This is the Δ
  - Four states: $I = 3/2$: $\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$
  - There is NO Δ^{--}

- In 1950's a new class of hadrons seen
  - Produced in πp interaction via Strong Interaction
  - But travel measureable distance before decay, so decay is weak

Why should this happen? There must be quantum number conserved in multihadron production that cannot be conserved in single hadron decay.
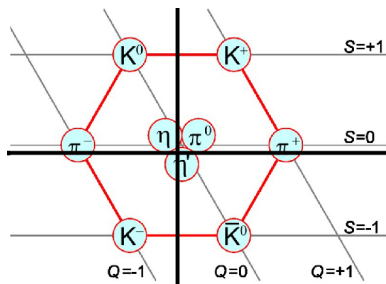
Strangeness (II)

- Example:

$$\pi^- p \to \Lambda^0 K^0$$

  - $\Lambda^0 \to p\pi^-$ with lifetime $\tau = 2.6 \times 10^{-10}$ sec
  - $K^0 \to \pi^+\pi^-$ with lifetime $\tau = 0.8958 \times 10^{-10}$ sec

- Assign a new quantum number called strangeness to the Λ and K^0

- By convention Λ has $S = -1$ and K^0 has $S = 1$ (an unfortunate choice, but we are stuck with it)

- Strangeness is an additive quantum number
  - Corresponds to a continuous symmetry
  - Can we extend the SU(2) ideas to include strangeness?
    - Natural choice: SU(3)

Group Theory Interpretation (SU(3))

0^{-+} (Pseudoscalar Mesons)

- Particles with same spin, parity and charge conjugation symmetry described as multiplet

  ▸ Different values of I_z and S and hence different values of charge and strangeness

- Raising and lowering operators to navigate around the multiplet

- Gell Man and Zweig: Patterns of multiplets explained if all hadrons were made of quarks

  ▸ Mesons: $q\bar{q}$ $3 \otimes \bar{3} = 1 \oplus 8$
  ▸ Baryons:
    qqq $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

- In those days, 3 flavors (extension to 6 discussed later)

Strangeness and I_Z

- We've already seen that within an isospin multiplet, different I_z have different charge
- Can generalize this observation for all light quark (u,d,s) multiplets:

$$Q = I_z + \frac{B + S}{2}$$

  Define hypercharge $Y \equiv B + S$

- This is called the Gell Mann-Nishijima Eq
- Note: Because Q depends on I_3, EM interactions cannot conserve isospin, but do conserve I_3
  - Analogous to the Zeeman effect, where a B field in z direction destroys conservation of angular momentum but leaves J_z as a good quantum number
- $\alpha = 1/137$ while $\alpha_S \approx 1$.
  - Effects of isospin non-conservation are small and can be treated as perturbative correction to strong interaction