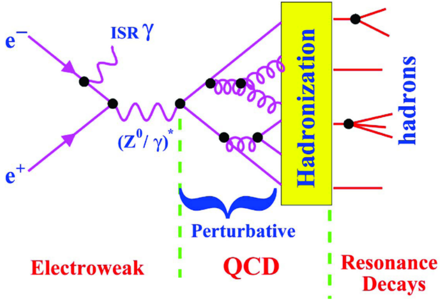
Lecture 12: QCD and $e^+e^- \rightarrow Hadrons$ Continued

Oct 4, 2016

Some material taken from Bill Gary's
2009 CTEQ summer school lectures

Reminder: $e^+e^- \to hadrons$

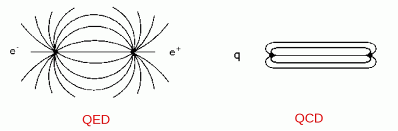


- Impulse approximation: Factorize process
  - short-distance hard scattering and long-distance fragmentation
- Lowest order hard scattering process is electroweak
  - Higer order QCD corrections
- Running of α_s means QCD description itself factorized
  - High q^2 perturbative (calculable)
  - Low q^2 hadronization (phenomenological model)

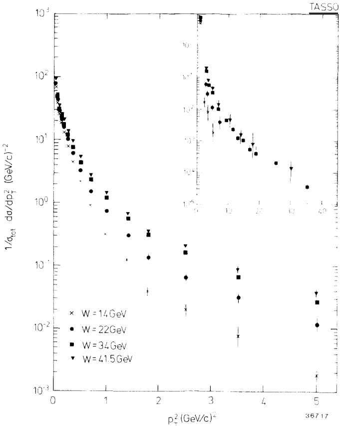
Today's game plan

- Continue discussion of hadronization
  - Does data agree with phenomenological picture?
- Add QCD corrections to hard scattering
  - Corrections to R
  - Three jet production
- Explore choices of Jet-finding algorithm
- Measuring α_S

Characterizing hadronization using e^+e^- data: Limited Transverse Momentum



- q and $\overline{q}$ move in opposite directions, creating a color dipole field
- Limited p_T wrt jet axis
  - $\sqrt{< p_T^2 >} \sim 350$ MeV
  - Well described by Gaussian distribution
- Range of longitudinal momenta (see next page)



SO [4.1] normalized differential cross section for the square of the momentum component transverse to the jet axis (= sphericity / = 14, 22, 34 and 41.5 GeV.

Characterizing hadronization using e^+e^- data: Rapidity and Longitudinal Momentum

- Define new variable: rapidity

$$y = \frac{1}{2} \ln \frac{E + p_{||}}{E - p_{||}}$$

- Phase space with limited transverse momentum:

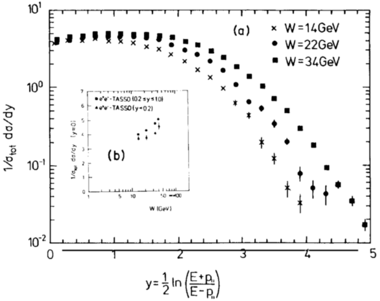
$$\frac{d^3p}{E} \to e^{-p_T^2/2\sigma^2} dp_T \frac{dp_{||}}{E}$$

- But

$$dy = \frac{dp_{||}}{E}$$

  (you will prove this on HW # 6)

- Rapidity is a longitudinal phase space variable



- Particle production flat in rapidity
- y_{max} set by kinematic limit $(E - p_{||}) \geq m_h$
- Height of plateau independent of $\sqrt{s}$
  - Multiplicity increase due to change in y_{max}

Hadronization: Particle Multiplicity

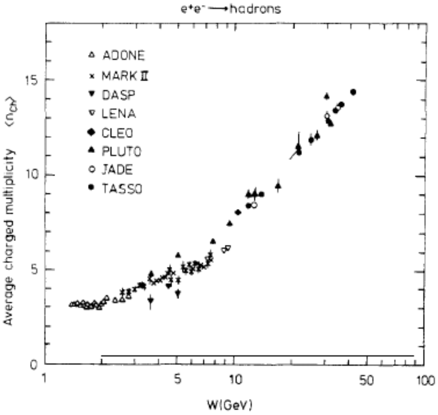


Fig. 4.1. Energy dependence of the average charged multiplicity.

- HW # 6 will include derivation of $< N_h >\sim \ln(\frac{E_{cm}}{m_h})$
- This expression holds for E_{cm} above a few GeV

Jet Structure Revisited: Reminder from Last Time

- Define Sphericity Tensor

$$M_{ab} = \sum_{i}^{N} p_{ia} p_{ib}$$

where a and b are x, y and z and the sum over i is a sum over all the (charged) particles in the event

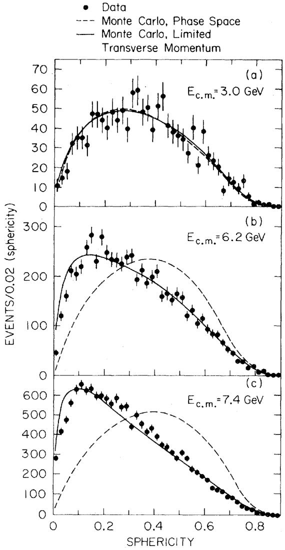
- Define the 3 normalized eigenvalues:

$$Q_k \equiv \Lambda_k / \sum_{i}^{N} p_i^2$$

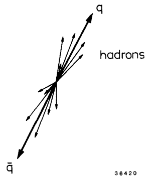
where Λ_k are the 3 eigenvalues of the matrix

- Principle axis $\hat{n}_3$ is jet direction
- Define the sphericity S

$$S = \frac{3}{2}(Q_1 + Q_2) = \frac{3}{2} \sum_i \frac{(p_{T,i}^2)_{min}}{\sum_i p_i^2}$$



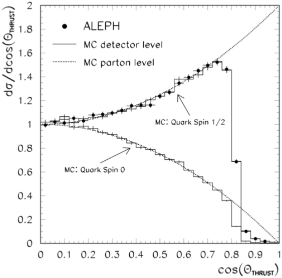
An alternative event shape variable: Thrust

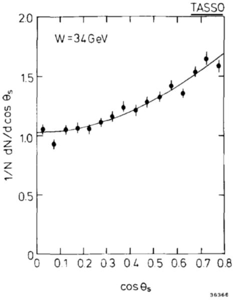


- Sphericity quadratic in p
  - Sensitive to hadronization details
- Linear alternative: Thrust axis

$$T = \max \frac{\sum |\vec{p}_i| \cdot \hat{n_T}}{\sum |\vec{p}_i|}$$

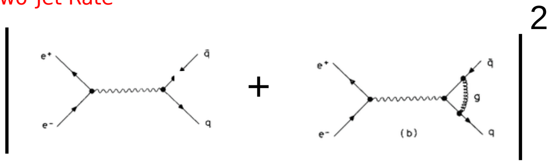
- Both choices appear to track quark direction well



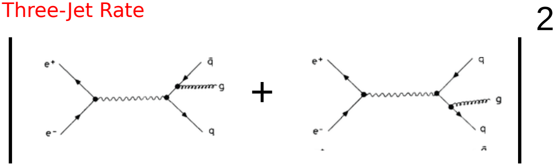


QCD corrections to $e^+e^- \rightarrow hadrons$

Two-Jet Rate



Three-Jet Rate



- Two- and Three-jet rates separately diverge
- Sum of the two converge (see next page)
- Can only define sensible three-jet rate with a cutoff in 3^{rd} jet energy

First Order QCD: Jet rates

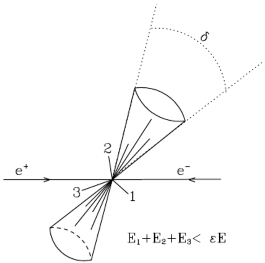
- Using gluon mass to regularize:

$$
\begin{array}{ll}
2 \text{ jet}: & \sigma_0 (1 + \frac{\alpha_s}{2\pi}\frac{4}{3}\left\{-\ln^2\left(\frac{m_g}{Q}\right) - 3\ln\left(\frac{m_g}{Q}\right) + \frac{\pi^2}{3} - \frac{7}{2}\right\} \\
3 \text{ jet}: & \sigma_0 \frac{\alpha_s}{2\pi}\frac{4}{3}\left\{\ln^2\left(\frac{m_g}{Q}\right) + 3\ln\left(\frac{m_g}{Q}\right) - \frac{\pi^2}{3} + 5\right\} \\
\text{Sum}: & \sigma_0 \left(1 + \frac{\alpha_s}{\pi}\right)
\end{array}
$$

(see Halzen & Martin pg 244)

- Cancellation of divergences not an accident
- Occurs throughout gauge theories (QED as well as QCD)
- Cancellation of infrared divergences described using general theorm by Kinoshita, Lee and Nauenberg
- In practice, divergences in 2 and 3 jet rates NOT a problem
  - Can only distinguish two jets if they are separated in angle and both jets have measurable energy.

How to define a 2-jet event: Sterman-Weinberg



- Classify as a two-jet event if we can find two cones of opening angle δ that contain all but at most a fraction ϵ of the total energy of the event
  - So, the classification depends on the values of δ and ϵ chosen
- In QCD theory, the jets are defined in terms of the partons of the calculation.
- In experiment, defined in terms of final state particles
  - Or in terms of proxies for these particles (eg energy clusters in a calorimeter)

Calculating the 3-jet rate in region away from singularity

- Define the energy fractions of the 3 jets

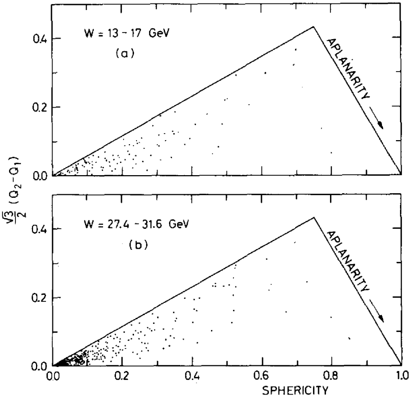
$$x_q = \frac{2E_q}{\sqrt{s}}; \quad x_{\overline{q}} = \frac{2E_{\overline{q}}}{\sqrt{s}}; \quad x_g = \frac{2E_g}{\sqrt{s}};$$

- Conservation of energy: $x_q + x_{\overline{q}} + x_g = 2$
- In practice, don't know which is the q, $\overline{q}$, g
- Order them in momentum

$$\frac{d\sigma_{3\ jet}}{dx_1 dx_2} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- Note: σ diverges if $x_1 \to 1$ or $x_2 \to 1$
  - $\vec{p_3} \,||\, \vec{p_1} \Rightarrow x_2 \to 1$: Collinear divergence
  - $x_3 \to 0 \Rightarrow x_1, x_2 \to 1$: Soft Divergence

Searching for 3 jet events using the Sphericity Tensor



$Q_1 + Q_2 + Q_3 = 1$

Sphericity $S = \frac{3}{2}(Q_1 + Q_2)$

Aplanarity $A = \frac{3}{2}Q_1$

- As the energy increases, the narrowing of the jets allows us to look for cases of wide angle gluon emission (3-jet events)
- QCD brem cross section diverges for colinear gluons or when the gluon momentum goes to zero
  - But that is the case where we can't distinguish 2 and 3 jet events anyway
  - Total cross section is finite (QCD corrections to R)
- Can use the sphericity tensor to search for 3-jet events
- Similar searches using a thrust-like variable possible: see next page

Thrust-like Energy Flow Method

- For each particle define an "energy flow vector"

$$\vec{E}_i = (E_i/|\vec{p}_i|)\, p_i$$

- Unit vector $\hat{e}_1$ analogout to Thrust T is:

$$F_{thrust} = \max \frac{\sum_i |\vec{E}_i \cdot \hat{e}_1}{\sum_i E_i}$$

- Orthogonal axes defined as

$$F_{major} = \max \frac{\sum_i |\vec{E}_i \cdot \hat{e}_2}{\sum_i E_i} \quad \hat{e}_2 \perp \hat{e}_1$$

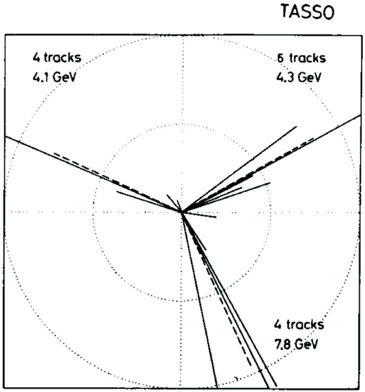
and

$$\hat{e}_3 = \hat{e}_1 \times \hat{e}_2$$

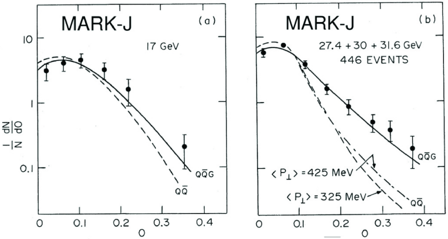
Global variables such as energy-flow and sphericity
are called "shape-variables"

Discovery of gluon jets: PETRA @ DESY

TASSO, PLB86(1979)243; MARK-J PRL43(1979)830; PLUTO PLB86(1979)418; JADE PLB91(1980)142



1st three-jet event seen by TASSO



Oblateness O = $T_{major} - T_{minor}$:

→ Events at E_{CM} ~30 GeV exhibit larger Oblateness (planar structure) than models without hard gluon radiation

Jet Finding Algorithms

- Shape variables like Thrust have advantage that they allow tests with minimal sensitivity to hadronization
- But don't allow us to study multijets well
- Need an algorithm to decide how many jets we have and associate particles with the jets
  - Algorithm will have some parameter to handle the infrared divergence (eg a cut-off)
- Two basic types of algorithm:
  - Geometric cluster algorithms:
    - Cluster based on angular separation. Define in terms of a cone-size (eg the δ of Sterman-Weinberg)
  - Recombination cluster algorithm
    - Find particles close together in a momentum-based metric and replace them with the sum of their four-momenta
- Traditionally, e^+e^- experiments used recombination algorithms and hadron colliders used geometric algoritms
  - LHC has moved to recombination algorithms as well (better behavour when comparing to theory)

What is important in a jet-finding algorithm?

- Should combine particles (or energy clusters) into jets in a way that agrees with what we see "by eye" in straightforward cases
  - Avoid pathologies (turns out this isn't easy)
- Should be insenstive to details of the hadronization
  - If a particle decays, calculation using parent and daughters should give nearly the same answer
- Should be possible to apply same algorithm to the quarks and gluons that are the outgoing "particles" in a QCD calculation (before hadronization)
  - Should not have divergences for colinear or soft emission: "Colinear and Infra-red safe"

The Basics of Recombination Cluster Algorithms

- Can start with any objects where we can define a 4-momentum, eg
  - Particles
  - Energy clusters

  Label them $i = 1 \ldots n$
- Loop over all these objects, calculating the distance between them according to a metric
- Combine the two that are closest together in that metric, if the distance is below a fixed cut
- A common metric: $y_{ij} = M_{ij}^2/s$ with $s = E_{CM}^2$
- What do we mean by "combining" the two? Different schemes:
  - E-scheme: Add 4-momenta $p_k = p_i + p_j$
  - EO-scheme: require jets to be massless

$$
\begin{aligned}
E_k &= E_i + E_j \\
\vec{p}_k &= \frac{\vec{p}_i + \vec{p}_j}{|\vec{p}_i + \vec{p}_j|} E_k
\end{aligned}
$$

- Iterate until all pairs satisfy $y_{ij} > y_{cut}$

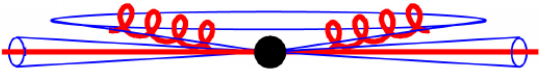
The JADE jet finder

JADE Collaboration (PETRA), Z. Phys. C33 (1986) 23

→ The original recombination jet algorithm

- Metric: $M_{ij}^2 = 2E_iE_j(1-\cos\theta_{ij}) \approx$ (invariant mass)^2
- Original version: E0-scheme combination of particles

Can lead to "**junk jets**":



→ a 2-jet event with soft, colinear radiation can be classified, unnaturally, as a 3-jet event

→ Inhibits NLLA re-summation techniques (what is 2-jets @ one order becomes >2-jets at higher order)

The k_T ("Durham") jet finder

S. Catani et al., Phys. Lett. B269 (1991) 432

- Metric: $M_{ij}^2 = 2 \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$
- E-scheme combination of particles

For small emission angles θ_{ij} ,

$$M_{ij}^2 \approx 2\min(E_i^2, E_j^2)[1-(1-\theta_{ij}^2/2+\cdots)] \approx \min(E_i^2, E_j^2)\theta_{ij}^2 \approx K_\perp^2$$

→ smaller of the transverse momentum of i wrt j vs. j wrt i

→ soft colinear radiation is attached to the correct jet

→ Largely inhibits junk jets, allows resummation

Note for future lectures

- The k_T algorithm works well in e^+e^- and is what was used for many of the results shown here
- Serious problems in hadron collisions due to sweeping up of soft particles from the proton remnants
- Turns out that changing the metric from

$$M_{ij}^2 = \min\left(E_i^2, E_j^2\right)\frac{R_{ij}}{D}$$

  to

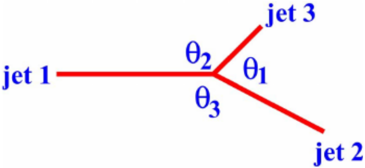
$$D_{ij} = \min\left(E_i^{-2}, E_j^{-2}\right)\frac{R_{ij}}{D}$$

  where R_{ij} is essentially θ_{ij} and D is essentially δ
- "anti-k_T algorithm" does wonders (more when we discuss jets at pp colliders)
- While k_T starts by combining softest particles, anti-k_T starts with the hardest ones
  - Less sensitive to pathologies from junk

3-jet matrix element: Spin of the gluon

Example: SLD Collaboration (SLC), PR D55 (1997) 2533

- **Select 3-jet events**: JADE jet finder with $y_{cut} = 0.02$
  → 25% of events classified as 3-jet events



$$E_i = E_{CM} \sin\theta_i / \sum_{i=1,3} \sin\theta_i$$

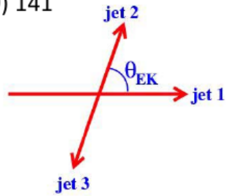
- **Calculate jet energies**: assume massless jets & E, p cons.

- **Order by energy**: $E_1 > E_2 > E_3$
  → jet 3 is the gluon jet in 75% of the events (energy tagging)
- **Scaled jet energies**: $x_i = 2E_i / E_{CM}$

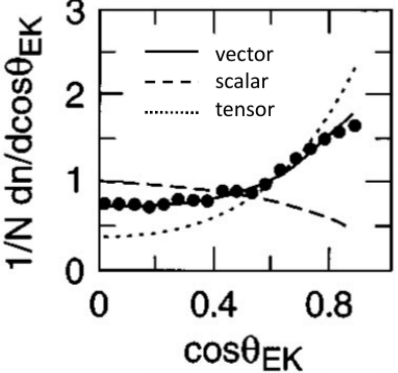
Ellis-Karliner angle $\cos\theta_{EK} = (x_2 - x_3) / x_1$

J.Ellis & I. Karliner, Nucl. Phys. B148 (1979) 141

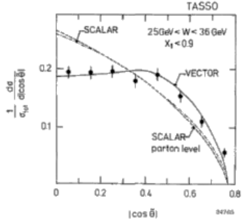
Scaled jet energies $x_i = 2E_i / E_{CM}$



SLD (SLC), PR D55 (1997) 2533



TASSO Collaboration (PETRA)
PL B97 (1980) 453



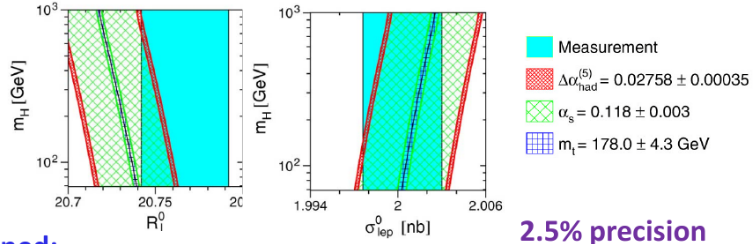
Measuring α_S: Overview

- Hadronization is always an issue in precision QCD measurements
- Best to cross-check results using as many different processes as possible
- In general, the more inclusive a measurement, the smaller the uncertainties
- Today, will talk about α_S determinations from LEP
- Last week, we saw that Deep Inelastic Scattering provides an alternative

Inclusive Measurements

- Based on event counting (independent of topology)
  - $R_\ell = \Gamma(Z \to hadrons)/\Gamma(Z \to \mu^+\mu^-)$
  - σ^0_{had} (peak hadronic cross section)
  - σ^0_{lep} (peak leptonic cross section)
  - $R_\tau = \Gamma(\tau \to hadrons)/\Gamma(\tau \to \mu\nu_\mu\nu_\tau)$

LEP & SLC Collabs., Phys. Rep. 427 (2006)257



- Recent reanalysis of these data gives

$$\alpha_S(M_Z^2) = 0.1196 \pm 0.0030$$

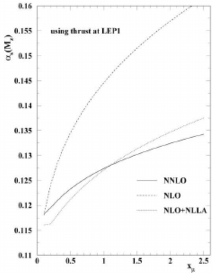
- Dominated by experimental uncertainty of 0.0028

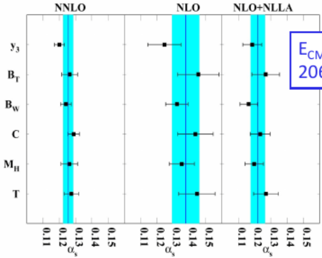
α_S from Event Shape Variables

- Larger systematic uncertainties due to hadronization
- Improvements in theory, but precision still theory-limited
- Some controversy about the quoted uncertainties

[Gehrmann-De Ridder et al., JHEP 12(2007)094]

→ Re-analysis of ALEPH data [G.Dissertori et al., JHEP 02(2008)040]





E_{CM}=91-206 GeV)

Renormalization scale uncertainty **reduced 30%** wrt α_S^2+NLLA

$$\boxed{\alpha_S(M_Z) = 0.1240 \pm 0.0013\,(\text{expt.}) + 0.0031\,(\text{theor.})}$$

→ **2.7% precision**